Chapter 22 - cost curves

Definitions:

→ Average cost = $\frac{c(w_1, w_2, y)}{y} = AC(y)$

→ cost per unit output

→ average variable cost = $AVC(y) = \frac{c_v(y)}{y}$

→ variable cost per unit output

→ average fixed cost = $AFC(y) = \frac{F}{y}$

→ the fixed costs per unit output

→ So, average cost = $AVC(y) + AFC(y)$

→ Marginal cost = $MC(y) = \frac{\partial c(y)}{\partial y}$

→ rate of change in costs for ~~another~~ a change in output

Example of deriving costs:

Let $C(y) = C_v(y) + F$

$= 10y^2 + 100$

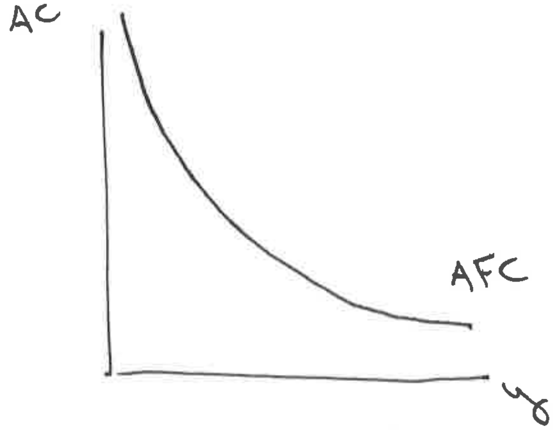
$$AC(y) = \frac{10y^2 + 100}{y} = 10y + \frac{100}{y}$$

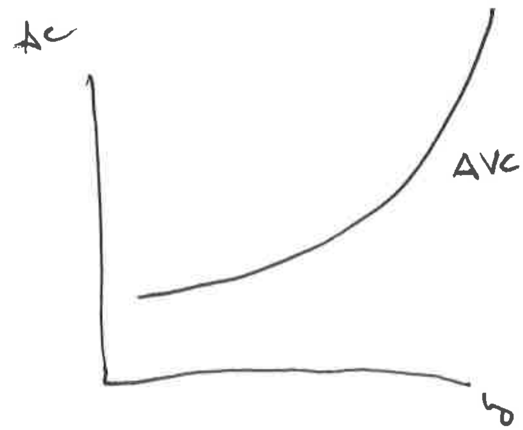
$$AVC(y) = \frac{C_v(y)}{y} = \frac{10y^2}{y} = 10y$$

$$AFC(y) = \frac{F}{y} = \frac{100}{y}$$

$$MC(y) = \frac{\partial C(y)}{\partial y} = 2(10y) = 20y$$

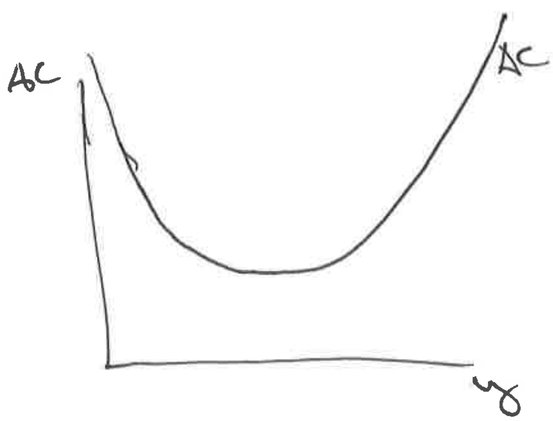
Representing the cost curves graphically:





→ Declining as spread fixed cost over more output

→ may decline initially or no, but increasing in y above some point b/c fixed factors

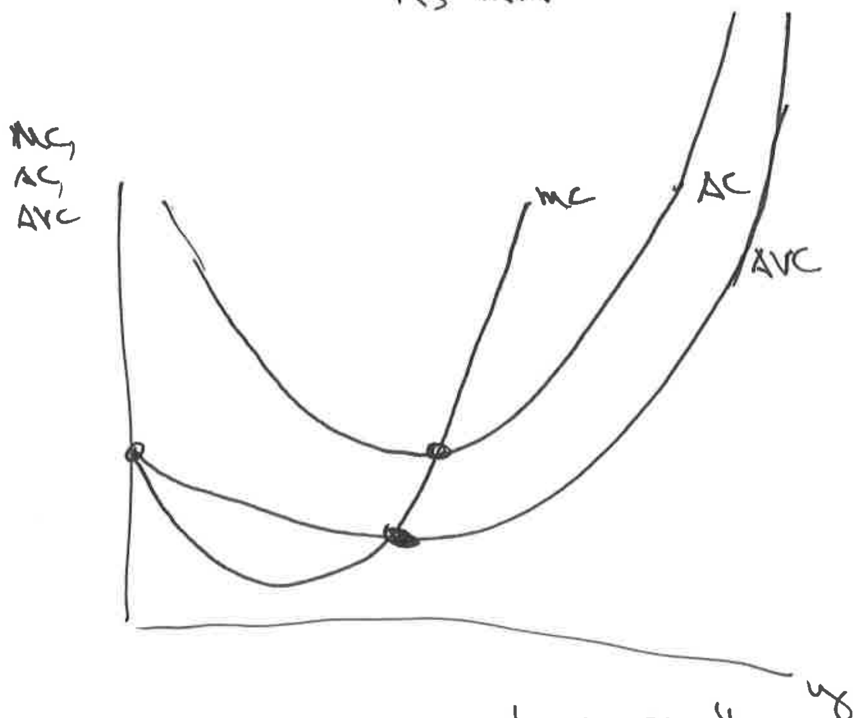


AC = AFC + AVC

→ How does marginal cost relate?

→ Remember, if MC < AC, then it pulls AC down as y↑. If MC > AC, it pulls AC up

⇒ MC intersects AC curve at its min



→ MC = AVC at very small levels of output

→ MC = AC at min AC

→ MC = AVC at min AVC

Example: Splitting output between 2 plants

→ diff cost functions $c_1(y_1)$, $c_2(y_2)$

problem:

$$\min_{y_1, y_2} \; c_1(y_1) + c_2(y_2) \quad \text{s.t. } y_1 + y_2 = y$$

$$\mathcal{L} = c_1(y_1) + c_2(y_2) - \lambda(y_1 + y_2 - y)$$

FOCs:

1) $\frac{\partial \mathcal{L}}{\partial y_1}$: $\frac{\partial c_1(y_1)}{\partial y_1} - \lambda = 0$

2) $\frac{\partial \mathcal{L}}{\partial y_2}$: $\frac{\partial c_2(y_2)}{\partial y_2} - \lambda = 0$

3) $\frac{\partial \mathcal{L}}{\partial \lambda}$: $y_1 + y_2 - y = 0$

(1) + (2) ⇒

$$mc_1 = \lambda = mc_2$$

⇒ y_1, y_2 solve

$$mc_1(y_1) = mc_2(y_2)$$

and

$$y_1 + y_2 = y$$

Long-Run Costs

→ in the long-run, all factors variable

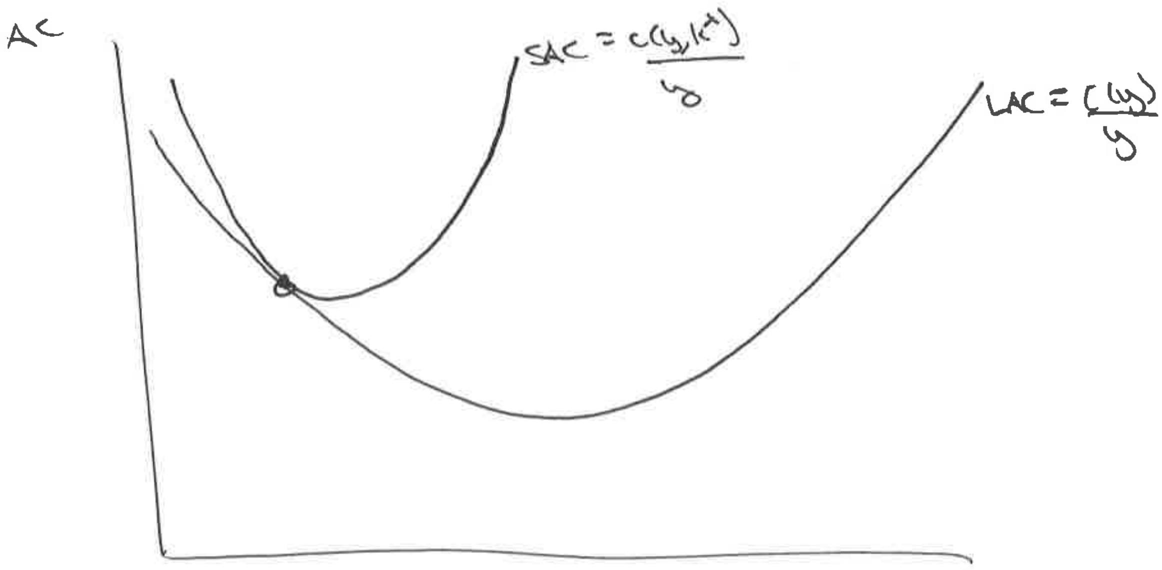
$$c(y) = c_s(y, k(y))$$

→ think about it ⇒ if can adjust all factors then LR costs must be less than or equal to the SR costs

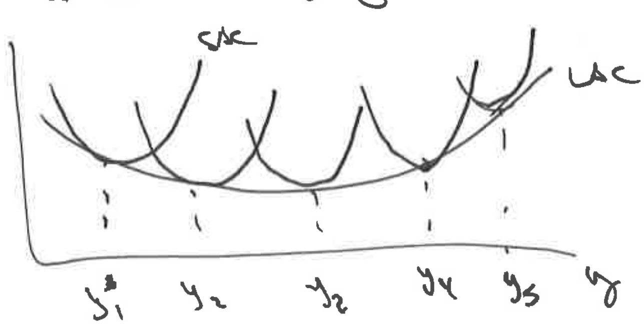
$$c(y) \leq c_s(y, k^*)$$

k^* LR choice of k (e.g. plant size)

$$c(y^*) = c_s(y^*, k^*)$$



at all levels of y (and ~~thus~~ thus k(y))



→ LAC curve the envelope of the SAC curves as vary y

→ Long-run MC?

$$\cancel{\text{Or}} \quad c(y) = c_s(y, k(y))$$

$$\frac{\partial c(y)}{\partial y} = \frac{\partial c_s(y, k(y))}{\partial y} + \frac{\partial c_s(y,k)}{\partial k}\frac{\partial k(y)}{\partial y}$$

→ evaluate at specific level of output, y^*, and associated optimal plant size $k^* = k(y^*)$, we know

$$\frac{\partial c_s(y^*, k^*)}{\partial k} = 0 \qquad \text{b/c } k \text{ } (\cancel{\text{k}} \text{ is optimally chosen})$$

$$\Rightarrow \frac{\partial c(y^*)}{\partial y} = \frac{\partial c_s(y^*, k^*)}{\partial y}$$

$$\Rightarrow \text{LR MC} = \text{SR MC}$$